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STEADY AND TRANSIENT FILM BOILING ON A SPHERE IN FORCED CONVECTION

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Abstract—A theoretical study of film boiling on a sphere in forced convection is given. First of all the steady state is analysed, that is, the assumption is made that the sphere temperature is maintained constant. Subcooling is included in the model. Simple analytical solutions are obtained for the vapour film thickness and local Nusselt number in two limiting cases, corresponding roughly to large and small subcooling. The usefulness of these two results is enhanced by two factors: the range not covered by either limiting case is rather small, and the parameters occurring in the solutions do not depend explicitly on the sphere radius or its velocity. Next it is shown that (in the parameter regime appropriate to the experiments so far reported) the transients in the vapour and coolant are negligible and therefore a quasi-steady approximation is valid in those regions. Thus the steady-state theory already obtained can be used to formulate the unsteady heat-transfer problem for the sphere. The solution of this problem is given.

NOMENCLATURE*

Coordinates and field variables

h,	vapour film thickness;
r, θ, λ,	spherical polar coordinates;
s, n,	tangential and normal coordinates in vapour film;
t,	time;
p,	pressure;
y,	boundary-layer coordinate in liquid, $\left(\frac{r}{a}-1\right)Pe^{1/2}$;
u, v,	velocity in vapour film;
U,	velocity in liquid;
T,	temperature;
δ_T,	(dimensionless) thickness of thermal boundary layer in liquid;
$f(\theta)$,	angular dependence of δ_T;
$g(\theta)$,	second approximation in vapour film (equation 29);
Q,	rate of production of vapour at angle $\theta/2\pi a^2 \sin\theta$.

Material constants (and geometrical constants)

a,	radius of sphere;
μ,	viscosity;
ν,	kinematic viscosity;
k,	thermal conductivity;
κ,	thermal diffusivity;
Δ_1,	$T_w - T_s$;
Δ_2,	$T_s - T_\infty$;
A_1, A_2,	coefficients in Pohlhausen solution;
L,	latent heat of vapourisation;
c_p,	specific heat of vapour.

Dimensionless quantities

Re,	Reynolds number in coolant, $U_\infty a/\nu_l$;
Pe,	Peclet number in coolant, $U_\infty a/\kappa_l$;
Pr,	Prandtl number in coolant, ν_l/κ_l;
T^*,	$(T-T_\infty)(T_s-T_\infty)$;
α,	$4\mu_g/U_\infty a\rho_l$;
β,	$16k_g\mu_g\Delta_1/3\rho_g\rho_l U_\infty^2 a^2 L$;
γ,	$16k_l\mu_g\Delta_2/3\rho_g\rho_l U_\infty^2 a^2 L \times \frac{2}{(\pi)^{1/2}} Pe^{1/2}$;
ε,	β^3/γ^4;
A,	$\alpha\gamma^2/\beta^2$;
δ,	$\alpha\beta^{-1/2}$;
b,	equation (35);
q,	$16\mu_g\beta/3\rho_l U_\infty^2\gamma^2$;
h^*,	h/a;
H,	$\frac{\gamma}{\beta}h^*$;
$\mathscr{H}$,	$\beta^{-1/4}h^*$.

Subscripts

w,	sphere surface;
s,	(for material constants) sphere;
s,	(for variables) at vapour/coolant interface;
g,	gas (i.e. vapour);
l,	liquid;
∞,	at ∞ in coolant;
Δ_0,	initial value of Δ_1;
U_f,	typical value in film.

INTRODUCTION

RECENTLY, considerable effort has been put in to understanding the phenomenon of vapour explosion, or transplosion as it is sometimes called, which sometimes occurs when very hot material, say a molten metal, is suddenly brought into contact with a liquid coolant. This might happen if the core of a nuclear reactor were to overheat and melt. If the hot material is in the form of coarse droplets to begin with, then normally each drop becomes surrounded

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*The Appendix is independent.

by a film of coolant vapour which strongly limits the rate of release of heat. As the drop cools, the film becomes unstable and there is a transition to nucleate boiling, with much greater heat-transfer rate. Occasionally the transition to rapid heat release occurs coherently over a large region of the mixture of hot and cool material and is accompanied by the break-up of the hot material into much finer particles. This is known as vapour explosion. It is important to understand this because the release of mechanical energy may have destructive effects.

The obvious starting point in the study is the analysis of film boiling on a single sphere. Much of the published literature on film boiling is concerned with free convection, that is, when the motion of the liquid coolant is produced by buoyancy forces and the hot object is held fixed. The problem of forced convection, when the hot object is moving through the coolant, is of greater relevance here, and in fact gravitational effects will be omitted altogether.

A detailed study of film boiling on a cylinder in forced convection has been given by Bromley, Leroy and Robbers [1]. Here the cylinder was held fixed in a stream of coolant at its saturation temperature. The appropriate theoretical analysis was derived; that is, a steady state was assumed, and heat transport within the liquid coolant was ignored. The analysis reduced eventually to the study of a single first order ordinary differential equation; however the structure of this equation is extremely inconvenient because the derivative appears non-linearly, and after a certain amount of *ad hoc* approximation the equation was solved numerically in particular cases. A similar analysis for the case of flow past a sphere was given by Kobayasi [2].

More recent experimental work, in which solid metal spheres were first heated and then passed rapidly through a tank of water, has been reported by Walford [3] and Stevens and Witte [4]; work with liquid sodium as coolant is described by Witte [5]. These experiments were designed in order to study the transition to nucleate boiling mentioned earlier, and are presumably intended to simulate the release of molten reactor fuel into liquid coolant. There are two principle differences between these experiments and those of Bromley *et al.* [1]. First, in most cases the liquid was significantly subcooled below its saturation temperature; this will have a marked effect on the vapour film because much of the heat released from the sphere will be convected away in the coolant without contributing to the production of vapour. Second, in most cases the stable film-boiling regime soon gave way to nucleate boiling, and indeed the prediction of the temperature at which this takes place is one of the major goals of the investigation. It follows that unsteady effects must be accounted for in the theory.

An attempt to provide an analysis for forced-convection film boiling on a sphere, including subcooling of the liquid and transient effects, was made by Hsaio, Witte and Cox [6]. However this analysis is unnecessarily complicated and contains several errors, of which the most important lies in the formulation of the heat-conduction problem for the metal sphere. We shall take up this question at a later stage.

The object of the present work is to provide the appropriate theory for the stable film-boiling regime in the experiments reported in [3] and [4]. Subcooling and unsteady effects will be incorporated, but no attempt is made to predict the conditions under which the vapour film gives way to nucleate boiling.

We begin by studying the steady-state film-boiling problem which would arise if the sphere temperature were maintained constant (but not necessarily uniform over its surface). This extends the work of Bromley *et al.* [1] and Kobayasi [2] to include subcooling. Furthermore, the equations will be recast in such a way as to make them much more amenable to analysis, and explicit analytical solutions will be given in two special cases, corresponding (roughly speaking) to large and small subcooling.

Although this theory is of some interest *per se*, its true importance is that it provides the key to the unsteady case. It will be shown that the transients in the liquid and vapour phase are negligible, in that they decay on a time scale much shorter than that of the heat release from the sphere. (This holds in the parameter ranges corresponding to the experiments.) In other words, the liquid and vapour react effectively instantaneously to changes in the sphere temperature. This means that a quasi-steady approximation will be valid in the liquid and vapour phases, and the appropriate theory is of course the steady-state theory already derived.

The rate of heat release from the sphere may now be calculated with the (unknown) sphere surface temperature occurring as a parameter. The final step is to use the result of this calculation to provide a boundary condition for the heat diffusion equation in the interior of the sphere, and thus determine the temperature distribution in the sphere. This problem now has the great simplification that the boundary condition does not depend explicitly on the time.

STEADY STATE ANALYSIS

A sphere of radius a and temperature T_w is held in a uniform stream of liquid which has undisturbed speed U_∞ and temperature T_∞. We suppose that T_w is held constant but may depend on θ. The temperature at the vapour–liquid interface is the saturation temperature T_s and this is supposed constant. We shall suppose that the thickness of the vapour film and of the thermal boundary layer in the coolant are small compared with a. (Thus, in particular, the Reynolds number of the coolant flow, $U_\infty a/\nu_l = Re$, is supposed large, and similarly the Peclet number $U_\infty a/\kappa_l = Pe$.) In these regions thin-film theory or boundary layer theory will be used.

Liquid region

We suppose that the velocity field is given by a potential flow. This seems to be the usual assumption made in the literature although not much explicit justification is given. The idea appears to be that the drag on the liquid is much reduced by the presence of the vapour film, which allows the liquid to "slip". We shall show that this idea is mistaken in general; in fact, the maximum velocity in the vapour film is much larger than U_∞, because the vapour is subject to the same pressure gradient as the liquid adjacent to it while having, of course, much lower density. The force exerted on the liquid is usually in the forwards or streamwise direction on the upstream side of the sphere. It is not possible to estimate this force at this stage because we have no *a priori* estimate of the film thickness; in the meantime we continue with the assumption of potential flow, in order to keep the exposition as simple as possible, and examine it in detail at the end of the analysis.

We give here only a sketch of the analysis since it may be obtained from [6] by deleting the unsteady terms. Then in spherical polar coordinates (r, θ, λ) centred on the sphere and with θ measured from the direction of the oncoming stream, the velocity components are

$$U_\infty\left[-\left(1-\frac{a^3}{r^3}\right)\cos\theta,\ \left(1+\frac{a^3}{2r^3}\right)\sin\theta,\ 0\right].$$

To find the temperature field we introduce a dimensionless boundary-layer coordinate $y = [(r/a)-1]Pe^{1/2}$, and then the (dimensionless) energy equation takes the form

$$\frac{3}{2}\sin\theta\frac{\partial T^*}{\partial\theta} - 3y\cos\theta\frac{\partial T^*}{\partial y} = \frac{\partial^2 T^*}{\partial y^2} \tag{1}$$

and the boundary conditions are

$$T^* = 1 \text{ on } y = 0, \quad T^* = 0 \text{ at } y = \infty.$$

Here, $T^* = (T-T_\infty)/(T_s-T_\infty)$. The solution of this problem is

$$T^* = \operatorname{erfc}[yf(\theta)] \tag{2}$$

where

$$f(\theta) = \frac{3}{4}\sin^2\theta\left(1-\frac{3}{2}\cos\theta+\frac{1}{2}\cos^3\theta\right). \tag{3}$$

A constant of integration has been determined by the requirement that $f(0) \neq 0$; in fact $f(0) = (3/2)^{1/2}$.

We shall require the temperature gradient at the interface, and this is given, in dimensional form, by

$$\frac{\partial T}{\partial r} = -\frac{\Delta_2}{a}\frac{2}{(\pi)^{1/2}}Pe^{1/2}f(\theta) = -\frac{\Delta_2}{a}\cdot\frac{1}{\delta_T} \tag{4}$$

where $\Delta_2 = T_s - T_\infty$ and δ_T is the dimensionless thickness of the thermal boundary layer

$$\delta_T = \frac{(\pi)^{1/2}}{2}Pe^{-1/2}\frac{1}{f(\theta)}. \tag{5}$$

In addition to the difficulty over the use of potential flow outlined above, the flow often separates forming a wake or cavity; the potential flow will of course apply only in the region where the flow remains attached.

Vapour region

Here we choose coordinates (s, n) where s is length measured around the sphere from $\theta = 0$ and n is length measured normal to the surface; thus $s = a\theta$ and $n = r-a$. The velocity components are (u, v). Making the usual boundary-layer approximations, the equation of continuity takes the form

$$\frac{1}{\sin\theta}\frac{\partial}{\partial s}(u\sin\theta)+\frac{\partial v}{\partial n} = 0 \tag{6}$$

and the equations of momentum and energy become

$$u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial n} = -\frac{1}{\rho_g}\frac{\partial p}{\partial s}+\nu_g\frac{\partial^2 u}{\partial n^2} \tag{7}$$

$$0 = -\frac{1}{\rho_g}\frac{\partial p}{\partial n} \tag{8}$$

$$u\frac{\partial T}{\partial s}+v\frac{\partial T}{\partial n} = \kappa_g\frac{\partial^2 T}{\partial n^2}. \tag{9}$$

The vapour–liquid interface is located at $n = h(s)$. The boundary conditions on u and T are

$$\begin{gathered} u = 0,\ T = T_w \text{ on } n = 0 \\ u = U_s = \frac{3}{2}U_\infty\sin\theta,\ T = T_s, \text{ on } n = h. \end{gathered} \tag{10}$$

The equations will be integrated from $n = 0$ to $n = h$, and this, together with certain boundary conditions, will enable us to eliminate v from the system. The continuity equation (6) gives

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}s}\sin\theta\int_0^h u\,\mathrm{d}n = \left(u\frac{\mathrm{d}h}{\mathrm{d}s}-v\right)_h = Q. \tag{11}$$

Here Q is the volume rate of evaporation of liquid at the location θ divided by $2\pi a^2\sin\theta$ and is coupled to the temperature field by means of the equation

$$Q = \frac{1}{\rho_g L}\left[k\frac{\partial T}{\partial n}\right]_h \tag{12}$$

in which the square bracket denotes the value of the discontinuity of the enclosed quantity at the interface $n = h$. This equation merely links the net flux of heat to the interface to the rate of evaporation, via the latent heat of vapourisation L. The temperature gradient on the liquid side has been given in equation (4), so that Q is in fact obtained in terms of vapour phase variables.

Equation (8) indicates that the pressure is independent of n, as usual in boundary-layer theory, and therefore equals $p(h)$, the value at the interface. By continuity of normal stress at the interface, this is equal to the pressure in the liquid at the interface, and this in turn may be found from the potential flow of the liquid by means of Bernoulli's equation. Thus

$$p = \text{constant} - \frac{1}{2}\rho_l\left(\frac{3}{2}U_\infty\sin\theta\right)^2. \tag{13}$$

Integrating (7) from $n = 0$ to $n = h$ and using (11) gives

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}s}\sin\theta\int_0^h u^2\,\mathrm{d}n - U_s Q = -\frac{1}{\rho_g}h\frac{\partial p}{\partial s}+\nu_g\frac{\partial u}{\partial n}\bigg|_0^h. \tag{14}$$

The first term on the left is the rate of change of momentum flux and the term U_sQ represents the momentum which is supplied by the freshly evaporated liquid. These terms will be omitted; the justification for this will be given later when an estimate of h is available.

A similar integration of the energy equation (9), again neglecting the analogous terms on the left, gives

$$0 = \kappa_g \frac{\partial T}{\partial n}\bigg|_0^h \tag{15}$$

so that the temperature profile is linear.

We now obtain an approximate solution of the equations by the usual Pohlausen-type integral technique. We set

$$u = A_1\frac{n}{h}+A_2\frac{n^2}{h^2} \tag{16}$$

$$T-T_s = \Delta_1\left(1-\frac{n}{h}\right) \tag{17}$$

where A_1 and A_2 are functions of s. Equation (17) already satisfies the boundary conditions and equation (15); applying the boundary condition at $n = h$ to (16) yields

$$A_1+A_2 = U_s \tag{18}$$

and substituting into (14) gives

$$0 = -\frac{1}{\rho_g}h\frac{\partial p}{\partial s}+\frac{2A_2 v_g}{h}. \tag{19}$$

From (12) we obtain

$$Q = \frac{1}{\rho_g L}\left(\frac{k_g\Delta_1}{h}-\frac{k_l\Delta_2}{a\delta_T}\right). \tag{20}$$

The continuity equation (11) becomes

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta\left(\frac{1}{2}A_1 h+\frac{1}{3}A_2 h\right) = Q. \tag{21}$$

This is the only differential equation remaining; A_1, A_2 and Q can be eliminated by means of (18), (19) and (20) and the equation then determines the film thickness $h(s)$. In terms of the dimensionless quantity $h^* = h/a$ it becomes

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta\,(h^{*3}\sin\theta\cos\theta+\alpha h^*\sin\theta) = \frac{\beta}{h^*}-\gamma f(\theta). \tag{22}$$

In this equation we have set

$$\alpha = \frac{4\mu_g}{U_\infty a\rho_l}$$

$$\beta = \frac{16}{3}\frac{k_g\mu_g\Delta_1}{\rho_g\rho_l U_\infty^2 a^2 L}$$

$$\gamma = \frac{16}{3}\frac{k_l\mu_g\Delta_2}{\rho_g\rho_l U_\infty^2 a^2 L}\frac{2}{(\pi)^{1/2}}Pe^{1/2}$$

where $f(\theta)$ is given in (3) and Δ_1 is supposed known, and in general depends on θ. Note that (13) has been used to determine the pressure gradient term in (19). This term was erroneously omitted in [6]; in fact it dominates the dynamics of the vapour layer as we shall see. We shall not discuss numerical values of α, β and γ at this point since the equation will be rescaled so as to introduce different constants.

Equation (21) cannot be integrated in closed form, but can, of course be integrated numerically in any particular case. However, this is of doubtful value, in view of the large number of parameters. Instead we shall show how the equation may be simplified in two special cases and then integrate it analytically. We shall thereby gain more insight into which terms are likely to be important. Furthermore, the simplified equations, as it turns out, have the great advantage that the dimensionless constants appearing in them are independent of U_∞ and a, and depend only on material constants, and Δ_1 and Δ_2. This greatly facilitates comparison with experiment. (The theory is not, of course, independent of U_∞ and a since it is assumed that $Re \gg 1$; but they do not appear explicitly.)

(i) *Large subcooling.* Here we begin by supposing that the LHS of (22) is negligible, and the main balance is between the two terms on the right. This expresses mathematically the situation in which almost all the heat arriving at the interface through the vapour is convected away in the liquid, and very little is used to produce vapour. This suggests that the order of magnitude of h^* is β/γ, and we therefore set

$$h^* = \frac{\beta}{\gamma}H. \tag{23}$$

After some rearrangement of (22) we find

$$\varepsilon\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta\,(H^3\sin\theta\cos\theta+AH\sin\theta) = \frac{1}{H}-f(\theta) \tag{24}$$

where

$$\varepsilon = \frac{\beta^3}{\gamma^4} = \frac{3\pi^2}{256}\frac{k_g^3\rho_g\rho_l\;\kappa_l^2 L}{k_l^4\mu_g}\frac{\Delta_1^3}{\Delta_2^4} \simeq 5\times 10^{-5}\frac{\Delta_1^3}{\Delta_2^4}$$

$$A = \frac{\alpha\gamma^2}{\beta^2} = \frac{16}{\pi}\frac{\mu_g k_l^2}{\rho_l k_g^2\kappa_l}\frac{\Delta_2^2}{\Delta_1^2} \simeq 1.6\times 10^2\frac{\Delta_2^2}{\Delta_1^2}.$$

As noted the constants are independent of U_∞ and a, and have been roughly evaluated for a water/steam system. We wish to neglect the LHS of (24) and this will be possible if ε is small and A is not large; it is easy to see that these requirements can be met simultaneously. In fact, if $\Delta_1 = 300°\text{C}$, Δ_2 need only be 10°C to give $\varepsilon = 0.13$, and A much less than unity. The largest value of Δ_1 reported by Stevens and Witte [4] is 146°; for this value $\Delta_2 = 6°\text{C}$ gives ε about 0.1. We may conclude that the large subcooling approximation $\varepsilon \ll 1$ typically requires only that Δ_2 be not less than 10°C or so.

The solution is simply

$$H = \frac{1}{f(\theta)} \tag{25}$$

to the first approximation.

A higher approximation in the form

$$H = \frac{1}{f(\theta)}[1+\varepsilon g(\theta)] \qquad (26)$$

may be sought and it is easy to show that

$$g(\theta) = \frac{\sin^2\theta + 4\cos^2\theta}{f^4} - \frac{4\cos\theta}{f^2} - \frac{4A}{3}. \qquad (27)$$

We note that $f(\pi) = 0$ so that H becomes infinite near $\theta = \pi$. This corresponds to the vapour film becoming separated in that region. Moreover the approximation (26) fails because $g(\theta)$ also becomes infinite; this indicates that the LHS of (24) is not uniformly negligible. This situation can be handled in principle by introducing new variables so as to restore the neglected terms to the equation. However, it turns out that this involves technical difficulties which it does not seem worthwhile to enter upon here, since in most cases the liquid flow will have separated before $\theta = \pi$ for some other reason, forming a wake, and this invalidates the whole basis for the theory. It should be mentioned, however, that the approximation (26) fails when $\pi - \theta$ is of order $\varepsilon^{1/8}$ which may not be particularly small.

(ii) *Small subcooling.* In this case we begin by neglecting the subcooling altogether, so that we set $\gamma = 0$ in (22). After some trial and error it is found that the main balance is between the first term in the bracket on the left, which derives from the pressure gradient, and the term in β/h^* on the right. This suggests that the order of magnitude of h^* is $\beta^{1/4}$, and so we set

$$h^* = \beta^{1/4}\mathscr{H} \qquad (28)$$

and then (22) is transformed into

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta\,(\mathscr{H}^3\sin\theta\cos\theta + \delta\mathscr{H}\sin\theta) = \frac{1}{\mathscr{H}} - \frac{f(\theta)}{\varepsilon^{1/4}} \qquad (29)$$

where ε is as before, and δ is given by

$$\delta = \frac{\alpha}{\beta^{1/2}} = \left(\frac{3\mu_g\rho_l L}{k_g\rho_l}\right)^{1/2}\frac{1}{\Delta_1^{1/2}} \simeq 1.1\times\frac{1}{\Delta_1^{1/2}}$$

where the numerical values are appropriate for water/steam. We are here supposing that ε is large, or rather that $\varepsilon^{1/4}$ is large. In the experiments of Walford [3] the largest value of ε occurs for $\Delta_1 = 800$°C, $\Delta_2 = 5$°C so that $\varepsilon^{1/4} \simeq 2.5$, so that the neglect of terms in $\varepsilon^{-1/4}$ is not really justified. However, since ε depends on Δ_2 so strongly, the case $\varepsilon \gg 1$ is not difficult to realise in practice. The term in δ is clearly negligible in most cases.

We consider the solution of (29) when the terms in ε and δ are omitted. We find

$$\mathscr{H}^4 = \frac{4}{3}(\sin\theta)^{-8/3}(\cos\theta)^{-4/3}\int_0^\theta (\sin\theta')^{5/3}(\cos\theta')^{1/3}\,\mathrm{d}\theta'. \qquad (30)$$

This solution expresses a balance between the production of vapour at the interface and the flow produced by the pressure gradient. This in turn is produced by the potential flow of the liquid and therefore vanishes at $\theta = \pi/2$. This is reflected in the singularity in (30), as a result of which $\mathscr{H}$ tends to infinity as $\theta \to \pi/2$.

In this region ($\theta \simeq \pi/2$) the neglect of the small term in δ is no longer justified; this is the term corresponding to the frictional drag of the coolant on the vapour film. It might be supposed that these forces might prevent, or at any rate significantly delay, the singularity in the film thickness. However, this is not the case.

To include these terms the equation must be rescaled near $\theta = \pi/2$. We begin by noting the precise form of the singularity in $\mathscr{H}$ as given by (30), which is

$$\mathscr{H} \sim b\left(\frac{\pi}{2}-\theta\right)^{-1/3} \qquad (31)$$

where

$$b = \frac{4}{3}\int_0^{\pi/2}(\sin\theta)^{5/3}(\cos\theta)^{1/3}\,\mathrm{d}\theta = \frac{2}{9}\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right). \qquad (32)$$

It may be shown that the appropriate variables near $\theta = \pi/2$ are

$$\phi = \delta^{-3}\left(\frac{\pi}{2}-\theta\right)$$
$$G = \delta\mathscr{H} \qquad (33)$$

and the equation, to leading order in the small quantity δ, becomes

$$\frac{\mathrm{d}}{\mathrm{d}\phi}(\phi G^3 + G) = 0$$

or

$$\phi G^3 + G = m. \qquad (34)$$

We see that $\phi G^3 \to m$ as $\phi \to \infty$ and comparing with (31) we have $m = b^3$. The solution of (34) is still singular because $\mathrm{d}G/\mathrm{d}\phi$ becomes infinite when $3\phi G^2 + 1 = 0$; combining this with (34) we find that the singularity is located at

$$\phi_0 = -3^9 2^{-4}\left[\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)\right]^{-6} = -0.540. \qquad (35)$$

Thus the separation of the vapour layer produced by the reversal of the pressure gradient is somewhat delayed by the drag of the coolant and occurs at $\theta = (\pi/2) - \phi_0\delta^3$. Since δ is small this angle will normally be insignificant.

Summary of results

The analysis given here gives the film thickness and the heat-transfer results are easily obtained. The temperature gradient at the sphere surface is

$$\left.\frac{\partial T}{\partial n}\right|_0 = -\frac{\Delta_1}{h} \tag{36}$$

so that the Nusselt number is essentially the reciprocal of the film thickness:

$$Nu = \frac{1}{h^*}. \tag{37}$$

For the case of large subcooling

$$\frac{1}{h^*} = \frac{\gamma}{\beta} f(\theta) = \frac{k_l \Delta_2}{k_g \Delta_1} \frac{1}{\delta_T}; \tag{38}$$

this expresses the fact that all the heat from the sphere is convected away in the liquid, and the thickness of the vapour film adjusts itself to the value at which this heat flux can be sustained.

For small subcooling

$$h^* = \beta^{1/4} \mathscr{H}(\theta). \tag{39}$$

The ratio of the Nusselt numbers in these two limiting cases is $\gamma\beta^{-3/4} f(\theta)\mathscr{H}(\theta) = \varepsilon^{-1/4} f(\theta)\mathscr{H}(\theta)$, where ε is to be evaluated with the larger value of Δ_2. Since in that case $\varepsilon \ll 1$ this ratio is large.

Between the two cases analysed here there will, of course, be a parameter regime in which all the terms in equation (22) are important, and then there is no alternative to numerical integration. This regime is, however, quite narrow; for example, if $\Delta_1 = 400°\text{C}$, the subcooling is large if $\Delta_2 > 10°\text{C}$ ($\varepsilon < 0.3$) and negligible if $\Delta_2 < 1°\text{C}$ ($\varepsilon^{-1/4} < 0.13$).

To show how the gap between the two limiting cases which have been studied may be filled, without numerical integration, we observe that the film thickness at $\theta = 0$ may be obtained as the root of a quartic polynomial equation. This has been solved numerically and the details are given in Appendix B.

Finally in this section we examine two assumptions made earlier, that the liquid velocity is given by a potential flow and that the non-linear transport terms in (14) and the corresponding energy equation (15) are negligible. First, the non-linear terms. It is necessary to estimate the order of magnitude of the velocity in the vapour film, and, as mentioned earlier, this is generally much larger than U_∞. The answer can be obtained by solving (18) and (19) for A_1 and A_2 or by direct order of magnitude calculations. For large subcooling we find that the dominant term gives the estimate

$$\frac{U_f}{U_\infty} \sim \frac{\mu_l}{\mu_g} \frac{1}{Pr_l} \left(\frac{k_g \Delta_1}{k_l \Delta_2}\right)^2 \simeq 0.1 \times \left(\frac{\Delta_1}{\Delta_2}\right)^2 \tag{40}$$

and for small subcooling,

$$\frac{U_f}{U_\infty} \sim \left(\frac{k_g \rho_l \Delta_1}{\rho_g \mu_g L}\right)^{1/2} \simeq 2.3 \times \Delta_1^{1/2}. \tag{41}$$

We see that the ratio is particularly large in the case of small subcooling; the reason is that the film thickness is largest in this case, reducing the size of the viscous drag term.

Now, the non-linear transport term on the left of (14) is of the order $U_f^2 h/a$, and the viscous term on the right is of order $\nu_g U_f/h$. The ratio is $U_f h^2/a\nu_g$. The "worst" case will be that of small subcooling when U_f and h are largest, h being of order $a\beta^{1/4}$; then the ratio in question is of order

$$\frac{k_g \Delta_1}{\mu_g L} \simeq 8 \times 10^{-4} \Delta_1.$$

Thus the non-linear terms may be appreciable but will not be important. For large subcooling we find that the ratio is

$$\frac{\nu_l}{\nu_g} \frac{\mu_l}{\mu_g} \frac{1}{Pr_l^2} \left(\frac{k_g \Delta_1}{k_l \Delta_2}\right)^4 \simeq 2 \times 10^{-7} \left(\frac{\Delta_1}{\Delta_2}\right)^4.$$

The term $U_s Q$ on the left of (14) may be estimated using (20) to provide an estimate for Q, and it is easily shown that this term is smaller than the non-linear transport term.

Next we examine the assumption of potential flow. If, due to viscous forces in the vapour, the liquid velocity at the interface departed significantly from potential flow, there would be a boundary layer in the liquid, adjacent to the interface, of thickness of order $aRe^{-1/2}$. Outside this boundary layer the velocity would still be given by potential flow, U_p say, and so the tangential stress exerted at the interface would be of order $\mu_l(U_p - U_s)Re^{1/2}/a$. The tangential stress on the vapour side is of order $\mu_g(U_f - U_s)/h$. Equating the two stresses and recalling that $U_p \sim U_\infty$ and that U_f/U_∞ is given (in the two cases) by (40) and (41), we find that $U_p \simeq U_s$ provided

$$\frac{\mu_g}{\mu_l} \frac{a}{h Re^{1/2}} \frac{U_f}{U_\infty} \ll 1. \tag{42}$$

For large subcooling this reduces to

$$Pr_l^{1/2} \frac{k_g \Delta_1}{k_l \Delta_2} \simeq 0.04 \frac{\Delta_1}{\Delta_2} \ll 1$$

and this in fact is the most severe restriction on the validity of the whole theory. However, it is met in most of the experiments of Stevens and Witte [4] and some of those of Walford [3], (others are covered by the small subcooling approximation). It may be remarked that the only alternative to the assumption of potential flow (other than direct numerical solution of the boundary layer equations) would be an integral technique similar to the one used here for the vapour film. This could not be expected to produce accuracy better than 10 or 15% and it is therefore reasonable to tolerate a similar error from the potential solution. The effect of this error will be confined to the detailed determination of the function $f(\theta)$ in the heat-transfer rate at the vapour–liquid interface. The thermal boundary-layer structure in the liquid has already been incorporated into this calculation and only the coefficient functions in (1) will change somewhat. The main conclusion then follows as before, namely, that the dynamics of the vapour film is virtually irrelevant and that its thickness adjusts so as to sustain the given heat-transfer rate.

For small subcooling (42) becomes

$$\left(\frac{k_g \rho_l \mu_g \Delta_1}{\rho_l L \mu_l^{1/2}}\right)^{1/4} \simeq 0.3\Delta_1^{1/4} \ll 1$$

and this means that the vapour drag cannot be neglected and has an important effect on the liquid flow. However the details of the liquid flow are not so important in this case because the heat transport in the liquid is negligible by assumption, and the most important mechanical effect of the liquid flow is due to the pressure gradient at the interface, which is the same as that due to potential flow. This pressure gradient is transmitted through the liquid boundary layer without change. Referring to equation (29) we see that the important terms are given correctly by potential flow. The interface speed U_s is so much less than U_f in this case that the vapour can be regarded as satisfying a no-slip condition at $n = h(s)$; put another way, A_1 and A_2 are much greater than U_s, so that (18) can be replaced by $A_1 + A_2 = 0$. Thus U_s drops out of the analysis except near the singularity in the solution (30) at $\theta = \pi/2$. Near $\theta = \pi/2$ the term δ in (29) enters and the θ-dependence of this term (namely $\sin\theta$) involves the assumption of potential flow. If this assumption fails the numerical value of ϕ_0 in (35) will be in error, but the qualitative conclusion remains, that the singularity is displaced only by a distance of order δ^3.

Finally in this section we note that as the sphere cools Δ_1 decreases and the small subcooling regime (if it exists at the start) must give way to the large subcooling regime, and from the estimates just given it is evident that the approximations actually improve as Δ_1 decreases, at any rate so long as (40) remains valid. [Equation (40) was derived retaining only the dominant term assuming $\Delta_1 \gg \Delta_2$ and fails when $\Delta_1 \sim \Delta_2$.] This question will be taken up at the end of the next section.

ANALYSIS OF TRANSIENTS

In this section we examine the time-dependent terms in three regions.

For the liquid coolant we must adjust equation (1) by adding a term $\partial T^*/\partial t_1$ to the left hand side; here $t_1 = tU_\infty/a$ is the appropriate dimensionless time. The solution of the resulting equation with the initial condition $T^* = 0$ is given in [6] and was originally derived by Chao [7]. The correct time scale is a/U_∞ as indicated; this is apparent from the explicit solutions given by these authors although it was not displayed as such. The unsteady solution (which we shall not reproduce here) decays into the steady state solution by means of factors $\exp(-3t_1)$.

This may be compared, on the other hand, with the time scale of the heat release from the sphere, which is a^2/κ_s since this is controlled by a diffusion equation. The ratio of the two time scales is

$$\frac{a/U_\infty}{a^2/\kappa_s} = \frac{\kappa_s}{aU_\infty} \tag{43}$$

and this is typically 10^{-2} or less, if the Reynolds number of the coolant flow is large, even if the sphere is made of a good heat conductor such as copper.

Note that this conclusion holds when the coolant is a liquid metal which, for given U_∞ and a, would have a considerably lower Peclet number than would water. The form of the transient terms as given in [6] and [7] involves factors $\exp(-3Pe\bar{\tau})$ where $\bar{\tau} = \kappa_l t/a^2$, suggesting that the decay is dependent on Pe, but this dependence is spurious and is a consequence of the wrong choice of time scale used to define $\bar{\tau}$.

The question of the transients in the vapour needs a little more analysis. Equation (11), which expresses conservation of mass, must be replaced by

$$\frac{\partial h}{\partial t} + \frac{1}{\sin\theta}\frac{\partial}{\partial s}\sin\theta\int_0^h u\,dn = Q. \tag{44}$$

Upon making the same transformations as before, we find, for the case of large subcooling, that (24) is replaced by

$$\frac{16}{3}\frac{\beta}{\gamma^2}\frac{\mu_g}{\rho_l U_\infty^2}\frac{\partial H}{\partial t} + \varepsilon M = \left[\frac{1}{H} - f(\theta)\right] \tag{45}$$

in which εM stands for the term on the left of (24). This equation can be used to determine the time scale. In fact if the term εM is dropped the equation can be solved explicitly, and this neglect of the θ-derivatives will be valid uniformly in the initial instants because H is the small then.

We have the equation

$$q\frac{\partial H}{\partial t} = \frac{1}{H} - f(\theta) \tag{46}$$

where

$$q = \frac{16}{3}\frac{\beta}{\gamma^2}\frac{\mu_g}{\rho_l U_\infty^2} \tag{47}$$

and the solution satisfying the initial condition $H(0) = 0$ is

$$[1 - Hf(\theta)]\exp[Hf(\theta)] = \exp\left[-\frac{t}{q}f^2(\theta)\right] \tag{48}$$

so that the time scale of the decay into the steady state solution $Hf(\theta) = 1$ is q as expected. The ratio of this to the time scale of the heat release in the sphere is

$$\frac{q\kappa_s}{a^2} \simeq \frac{k_g \rho_g L \kappa_s \Delta_1}{k_l^2 Pe \Delta_2^2} \simeq \frac{10\Delta_1}{Pe\Delta_2^2} \tag{49}$$

and this number is small in the experiments with water, for which Pe is large, and the sphere was made of copper. The case of ultimate practical interest involving liquid metal coolants, for which Pe is not so large, may not be so large, may not be so clear cut; but here the sphere will probably consist of some relatively poor conductor of heat such as uranium dioxide, and also the value of Δ_2 will be large, so that it will probably be reasonable to assume that the ratio in (49) is small.

For the case of small subcooling the film thickness must be rescaled by means of the equation $H = \varepsilon^{-1/4}\mathscr{H}$, using (23) and (28), and although the equation cannot now be integrated with respect to t, because the θ-derivatives are present, it is clear that the time scale of the decay into the steady state is $q\varepsilon^{-1/4}$ and so instead of (49) we have

$$\frac{q\kappa_s}{a^2}\frac{1}{\varepsilon^{1/2}} = \frac{\kappa_s}{Re}\left(\frac{16}{3}\frac{\rho_g^3 L}{k_g \mu_g \rho_l \Delta_1}\right)^{1/2} \simeq \frac{3}{Re\Delta_1^{1/2}}$$

which is again very small.

The calculations given here show that if the unit of time is chosen to be a^2/κ_s, so that in the heat diffusion equation in the sphere the coefficients will be unity (choosing a as the unit of length, of course), then the coefficients of the transient terms in the coolant and in the vapour will be the small parameters given in (43) and (49). This suggests that a quasi-steady approximation will be appropriate in those regions; that is, we omit the unsteady terms, and the solutions are the steady-state solutions given in the previous section. Supposing that Δ_1 is given, these solutions enable us to calculate the rate of heat transfer from the sphere, and hence to formulate and solve the heat conduction problem in the sphere. From this we can calculate Δ_1 as a function of t, and hence finally verify *a posteriori* that the transients in the vapour and liquid are negligible.

The equation to be solved in the interior of the sphere is

$$\kappa_s\left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)\right] = \frac{\partial T}{\partial t} \tag{50}$$

and we suppose that initially the sphere is at uniform temperature T_0.

In the case of large subcooling, the boundary condition on $r = a$ is that of prescribed heat flux, given by (39) and (41):

$$k_s\frac{\partial T}{\partial r} = -k_l\frac{\Delta_2}{a}\frac{1}{\delta_T} = -\frac{k_l\Delta_2}{a}\frac{2Pe^{1/2}}{(\pi)^{1/2}}f(\theta)$$

$$\text{on} \quad r = a. \tag{51}$$

As anticipated this has the great simplification of being independent of t, but on the other hand is complicated by the θ-dependence. (This problem is incorrectly formulated in [6]).

The solution of (50) with its initial and boundary conditions is long and complicated; an outline is given in Appendix A. Here we give the result for the temperature at $r = a$, $\theta = 0$, that is, at the forward stagnation point; it is here that the temperature is usually measured. This is given by

$$\frac{T_0 - T}{T_0 - T_s} = \frac{k_l}{k_s}\frac{\Delta_2}{\Delta_0}\frac{2}{(\pi)^{1/2}}Pe^{1/2}\psi(t^*) \tag{52}$$

where $\Delta_0 = T_0 - T_s$, which is the initial value of Δ_1, $t^* = \kappa_s t/a^2$ is the appropriate dimensionless time, and the form of the function ψ is

$$\psi = 2.1213t^* + 0.6931$$
$$-0.2413\exp(-4.3480t^*)\ldots(t^* \text{ large}) \tag{53}$$

$$\psi = \left(\frac{6}{\pi}\right)^{1/2}t^{*1/2} + \frac{1}{2}6^{1/2}t^*$$
$$+\frac{4}{9}\left(\frac{6}{\pi}\right)^{1/2}t^{*3/2} + \frac{1}{2}6^{1/2}t^{*2}\ldots(t^* \text{ small}) \tag{54}$$

as explained in the Appendix. A graph of $\psi(t^*)$ is given in Fig. 1.

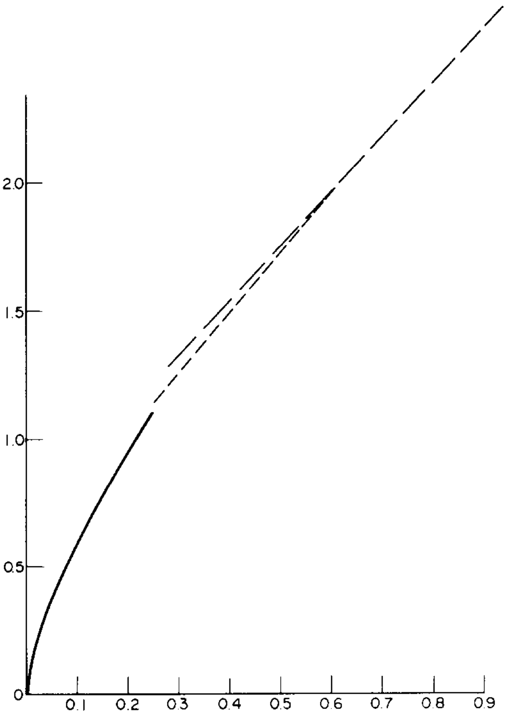

FIG. 1. Graph of the function $\psi(t^*)$ in equation (52):
——— small t^* expansion, equation (54);
--------- large t^* expansion, equation (53);
– – – asymptote as $t^* \to \infty$.

From (52) it is clear that significant change in T takes place in a time of order

$$\frac{a^2}{\kappa_s}\frac{k_s}{k_l}\frac{\Delta_0}{\Delta_2}Pe^{-1/2}$$

and it is this, rather than a^2/κ_s, which should be compared with the time scales in the coolant and vapour. Then the right hand sides of (43) and (49) should be multiplied by the factor $k_l Pe^{1/2}\Delta_2/k_s\Delta_0$ and even though Pe is large this ratio will generally be small. For a copper sphere the ratio k_l/k_s is about 1.6×10^{-3}. This completes the justification of the neglect of transients in the vapour and liquid.

The case of small subcooling presents considerably more difficulty in the evaluation of the sphere temperature, and we shall not enter into this problem here. Firstly the nature of the flow beyond $\theta = \pi/2$ is largely unknown because of the separation of the vapour film, which presumably forms a wake.

Thus no accurate heat-transfer data are available in that region. Secondly, even on the front portion of the sphere the form of the boundary condition, which is found from (42), is non-linear because of the occurrence of the factor $\beta^{1/4}$ (which contains Δ_1). However the lack of theory for this case is not so serious because the small-subcooling regime cannot persist for long. To see this we consider the form of ε, which is proportional to Δ_1^3/Δ_2^4. If initially Δ_1 is large enough to make ε large, which is the small subcooling approximation, as the sphere cools Δ_1 falls in magnitude and ultimately ε becomes small, which is the large subcooling approximation. For this reason the large subcooling theory is much the more important.

CONCLUDING REMARKS

It has been shown that transient film boiling on a sphere can, in the parameter ranges of interest, be reduced to a study of the steady-state equations in the vapour and liquid regions, together with an unsteady heat conduction problem in the solid sphere. As a consequence of the quasi-steady approximation valid in the vapour and coolant, the rate of heat transfer is determined solely by material constants and the Peclet number of the coolant, and in the important case of large subcooling, does not depend on the sphere temperature or (explicitly) on the time. The thickness of the vapour film adjusts itself continually to maintain the constant heat transfer rate as the sphere cools.

The thickness of the vapour film has been found in two limiting cases, those of large and small subcooling, and it happens that the range over which neither approximation holds is rather narrow—about 10°C. In the case of large subcooling the vapour film extends over the entire sphere, separating only at the rear stagnation point. For small subcooling the vapour film thickness tends to infinity (that is, separates) shortly after $\theta = \pi/2$.

Both these results are based on the assumption that the coolant flow is described by inviscid potential flow, that is, unseparated flow, the idea being that the force on the coolant exerted by the vapour will not be significant. This assumption may not be justified for reasons already discussed and may also fail when the film becomes extremely thin. We recall that initially the stress exerted by the vapour on the coolant is in the streamwise direction since $U_f/U_\infty > 1$. As the sphere cools, i.e. Δ_1 decreases, U_f decreases and eventually U_f/U_∞ becomes of order unity. After this the stress is reversed and retards the liquid, and we may enquire whether this might provoke separation of the coolant flow. We can estimate the time taken for U_f/U_∞ to decrease to order unity, which, from (40), occurs when $\Delta_1 \sim \Delta_2$. Using (52) to find Δ_1 as a function of t^*, we find

$$\Delta_2 \sim \Delta_0 - \frac{k_l}{k_s} \frac{2\,Pe^{1/2}}{(\pi)^{1/2}} \Delta_2 \psi(t^*). \tag{53}$$

Since $\Delta_2 \ll \Delta_0$ the balance can only be achieved when the terms on the right are of the same order.

This gives an estimate for t^*, which is therefore simply the time taken for the vapour film to disappear altogether. Assuming that ψ may be replaced by its leading term (as $t^* \to \infty$) we find

$$t^* \sim \frac{1}{2.1213} \frac{k_s}{k_l} \frac{(\pi)^{1/2}}{2\,Pe^{1/2}} \frac{\Delta_0}{\Delta_2}.$$

If typical values from the experiments are substituted into this expression it is found that the times are much longer than the time taken for the transition to turbulent film boiling or nucleate boiling, and usually longer than the entire experiment. For example, taking values from the experiments of Walford [3], $\Delta_0 = 700°\text{C}$, $\Delta_2 = 40°\text{C}$, $Pe = 2.4 \times 10^4$, gives $t = 2.6\,\text{s}$, compared with the duration of the experiment which was about 0.25 s. These results suggest that the breakdown of the film boiling regime is not likely to be linked with the separation of the liquid flow caused by vapour drag.

A detailed comparison of theory and experiment is not possible at the present time because of a lack of relevant data. Walford [3] gives only heat flux measurements averaged over the whole cooling process, including (in almost all cases) regimes other than stable film boiling. No data concerning the temperature as a function of the time are given. Stevens and Witte [4] measured the temperature as a function of time but give only a single oscilloscope trace on a scale which is too small to be of use in the present context. They also give instantaneous heat transfer rates calculated from the measured temperature traces by assuming the sphere cooled uniformly. This assumption is not justified, as the present analysis shows; it is possible in principle, however, to reconstruct the measured temperature gradient dT/dt from the data, but their use of log–log plots makes this difficult to do with satisfactory accuracy. A further difficulty arising from the fact that the time variation is not given explicitly is that in order to substitute into the function $\psi(t^*)$ it is essential to know whether t^* is large or small, since this obviously has a profound effect on the slope. [It may be remarked that the accuracy of the expressions for $\psi(t^*)$ is made worse if they are differentiated term by term.] However, assuming that $d\psi/dt^*$ lies between 2 and 5, and using (52) to substitute into Stevens and Witte's expression for the heat-transfer rate (the unnumbered formula on p. 447) gives values of the right order, namely about 6 (in their units).

No attempt has been made to predict or explain the transition to nucleate boiling or turbulent film boiling, or how these phenomena are linked to vapour explosions. The stable film-boiling regime analysed here may be regarded as providing the initial conditions for these studies.

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APPENDIX A

It is easy to see that the heat-diffusion problem in the sphere reduces to the following dimensionless boundary-value problem:

$$\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)=\frac{\partial T}{\partial t} \qquad \text{(A1)}$$

with

$$\begin{aligned}&\frac{\partial T}{\partial r}=f(\theta) \quad \text{on} \quad r=1,\ t>0\\ &T=0 \quad \text{when} \quad t=0.\end{aligned} \qquad \text{(A2)}$$

The function $f(\theta)$ is given in equation (3). To obtain a solution by separation of variables we begin by expressing $f(\theta)$ as a sum of Legendre polynomials:

$$f(\theta)=\sum_0^\infty A_nP_n(\cos\theta).$$

It is not necessary to find more than the first few coefficients A_n (as will be shown), which are

$$A_0=\frac{1}{2}(2)^{1/2}=0.7071,$$

$$A_1=\frac{9}{4}(2)^{1/2}\left[\frac{4}{5}(3)^{1/2}-\frac{6}{5}\right]=0.5907,$$

$$A_2=\frac{15}{2}(2)^{1/2}\left[\frac{62}{21}-\frac{12}{7}(3)^{1/2}\right]=0.0893,$$

$$A_3=\frac{21}{4}(2)^{1/2}\left[\frac{24}{5}(3)^{1/2}-\frac{364}{45}\right]=1.6702.$$

By expanding T in a similar fashion

$$T=\sum_0^\infty T_n(r,t)\,P_n(\cos\theta)$$

we obtain a sequence of problems for each T_n. We find

$$T_0=A_0\left[\frac{1}{2}r^2+3t-\frac{3}{10}-\frac{2}{r}\sum_{p=1}^{\infty}\frac{\sin a_{0p}r}{a_{0p}^2\sin a_{0p}}\exp(-a_{0p}^2t)\right] \qquad \text{(A3)}$$

$$T_1=A_1P_1(\cos\theta)\times\left[r-2\sum_1^\infty\frac{j_1(a_{1p}r)}{(a_{1p}^3-2a_{1p})j_1(a_{1p})}\exp(-a_{1p}^2t)\right] \qquad \text{(A4)}$$

and in general

$$T_n=\frac{A_n}{n}P_n(\cos\theta)\{r^n-\Sigma\}$$

Here j_n represents the spherical Bessel function of order n and a_{np} is the pth zero of j_n' (see for example Abramowitz and Stegun [8]).

Clearly if t is large enough all the infinite series may be neglected and we obtain for the value of T at $r=1$, $\theta=0$ (that is, at the forward stagnation point)

$$T=3A_0t+\frac{1}{5}A_0+\sum_1^\infty\frac{1}{n}A_n\ldots. \qquad \text{(A5)}$$

Although the A_n cannot be found conveniently it is possible to show (by an extremely lengthy calculation. for which the author is indebted to Mr. E. J. Watson) that

$$\sum_1^\infty\frac{1}{n}A_n=\frac{1}{2}(2)^{1/2}\left[(2)^{1/2}\sin^{-1}\left(\frac{2}{3}\right)^{1/2}-\frac{1}{3}+\log\frac{(3)^{1/2}+1}{2(3)^{1/2}}\right]=0.5517.$$

Next we examine the range of t over which the infinite series are negligible. In (A4) we find that the first value of a_{0m} is $a_{01}=4.4934$ so that the exponential terms tend to zero extremely rapidly as t increases. The whole series in fact contributes only 3% of the total when t is as small as 0.1. In (A5) the numbers are not so fortunately disposed since $a_{11}=2.08518$. If $t=0.2$ it will be necessary to retain the first term in this series, but since $a_{12}=5.94037$ the rest may be neglected. A similar inspection of the rest of the series shows that they may be neglected for $t\simeq 0.2$; for T_2 the approximation is improved by the circumstance that A_2 is small. We may improve (A5) somewhat, then, to

$$\begin{aligned}T&=3A_0t+\frac{1}{5}A_0+\sum_1^\infty\frac{1}{n}A_n-2A_1\frac{1}{a_{11}^3-2a_{11}}\exp(-a_{11}^2t)\\ &=2.1213t+0.6931-0.2413\exp(-4.3480t)\end{aligned} \qquad \text{(A6)}$$

and as explained this will give good accuracy down to about $t=0.2$.

Although the series (A4), (A5) give the solution for all t the convergence is poor for small t and the solution for small t is best obtained by another method. Since the temperature at $r=1$ is the quantity of interest the most convenient procedure is to introduce the similarity variable $\eta=(1-r)/t^{1/2}$ into the equation and boundary conditions and expand the solution in a power series in $t^{1/2}$:

$$T=t^{1/2}T^0(\eta,\theta)+tT^1(\eta,\theta)+t^{3/2}T^2(\eta,\theta)\ldots. \qquad \text{(A7)}$$

The θ dependence occurs only parametrically and so the equations for the coefficient functions are effectively ordinary differential equations. It is a straightforward but lengthy matter to obtain the appropriate solutions, which are

$$T^0=\tau_0(\eta)f(\theta),\quad \tau_0=2\,i\,\mathrm{erfc}\,\frac{1}{2}\eta$$

$$T^1=\tau_1(\eta)f(\theta),\quad \tau_1=-4\,i^2\,\mathrm{erfc}\,\frac{1}{2}\eta-2\tau_0'$$

$$T^2=[32f(\theta)-8g(\theta)]\,i^3\,\mathrm{erfc}\,\frac{1}{2}\eta+[g(\theta)-2f(\theta)]\tau_0-2\tau_1'f(\theta)$$

$$\begin{aligned}T^3&=[120g(\theta)-240f(\theta)]\,i^4\,\mathrm{erfc}\,\frac{1}{2}\eta\\ &\quad+[g(\theta)-4f(\theta)]\tau_1+[2g(\theta)+2f(\theta)]\,\eta\tau_0\\ &\quad+[4g(\theta)-12f(\theta)]\tau_0'-2\frac{\partial}{\partial\eta}T^2.\end{aligned}$$

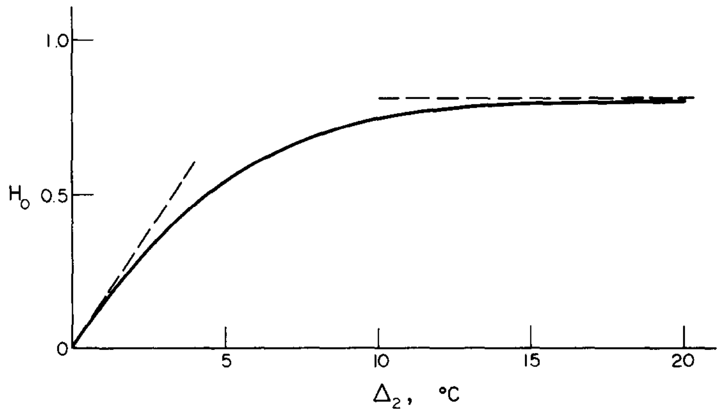

FIG. 2. Graph of the positive real root of ($B1$) with $\Delta_1 = 250$°C, as a function of Δ_2: ——— exact solution; -------- small subcooling approximation; — — — large subcooling approximation.

Here

$$g(\theta) = \frac{1}{\sin\theta}\frac{d}{d\theta}\sin\theta f(\theta).$$

By setting $\theta = 0$, $\eta = 0$ we find the temperature at the stagnation point as a function of t:

$$T = \left(\frac{6}{\pi}\right)^{1/2} {}^{1/2}t + \frac{1}{2}(6)^{1/2}t + \frac{4}{9}\left(\frac{6}{\pi}\right)^{1/2} t^{3/2} + \frac{1}{2}(6)^{1/2}t^2 \ldots \quad \text{(A8)}$$

which complements (A6). Graphs of the functions given by these equations are shown in the diagram, and it will be seen that a satisfactory merging of the two solutions is obtained.

APPENDIX B

We consider the solution of (27) near $\theta = 0$. Since $(dH/d\theta) = 0$ at $\theta = 0$ it is easy to show that $H(0) = H_0$ satisfies the equation

$$2\varepsilon(H_0^3 + AH_0) = \frac{1}{H_0} - f(0). \quad \text{(B1)}$$

Approximate values of ε and A are given after (27) with Δ_1 and Δ_2 left free. It is therefore a simple matter to compute the value of H_0 for any choice of Δ_1 and Δ_2, and this will be taken as representative of H. Our purpose here is to compare the exact value found in this way with the asymptotic solutions, valid for large and small subcooling, given in the main text.

As an illustration we set $\Delta_1 = 250$°C and computed the positive real root of (B1) as a function of Δ_2 from $\Delta_2 = 0$°C to $\Delta_2 = 20$°C, and the results are shown graphically in Fig. 2, along with the asymptotic solutions. Note that H_0 contains scaling factors which depend on Δ_1 and Δ_2, so that the film thickness does not really tend to zero as $\Delta_2 \to 0$. This diagram is intended to indicate the range in which the asymptotic solutions are useful rather than the actual variation in film thickness; a similar equation to (B1) involving h^* can of course be solved but this would lose the advantage, mentioned in the main text, that the coefficients in (B1) are independent of U_∞ and a.

EBULLITION EN FILM STATIONNAIRE ET TRANSITOIRE SUR UNE SPHERE ET EN CONVECTION FORCEE

Résumé—On étudie théoriquement l'ébullition en film sur une sphère en convection forcée. On analyse en premier le cas stationnaire, en supposant que la température de la sphère est maintenue constante. Le cas du sous-refroidissement est inclus dans le modèle. Des solutions analytiques simples sont obtenues pour l'épaisseur du film de vapeur et le nombre de Nusselt local dans deux cas limites qui correspondent au sous-refroidissement très grand et petit. L'utilité de ces deux résultats est accrue par deux facteurs: le domaine non couvert par ces cas limites est petit et les paramètres intervenant dans les solutions ne dépendent pas explicitement du rayon de la sphère ou de sa vitesse. On montre ensuite que (dans des conditions expérimentales qui seront décrites ultérieurement) les phénomènes transitoires dans la vapeur et le liquide sont négligeables et, par conséquent, on peut supposer valable une description quasi-stationnaire dans ces régions. Ainsi la théorie quasi-stationnaire obtenue peut être utilisée pour formuler le problème non stationnaire pour la sphère. On donne la solution de ce problème.

STATIONÄRES UND INSTATIONÄRES FILMSIEDEN AN EINER KUGEL BEI ERZWUNGENER KONVEKTION

Zusammenfassung—Es wurde eine theoretische Untersuchung des Filmsiedens an einer Kugel bei erzwungener Strömung durchgeführt. Zuerst wurde unter der Annahme, daß die Oberflächentemperatur konstant gehalten wird, der stabile Zustand untersucht. Unterkühlung ist in das Modell mit einbezogen. Einfache analytische Lösungen wurden für die Dicke des Dampffilms und die örtliche Nusselt-Zahl in zwei Grenzfällen erzielt, die etwa starker und geringer Unterkühlung entsprechen. Die Brauchbarkeit dieser beiden Ergebnisse wird erhöht durch zwei Faktoren: der Bereich, der von einem der beiden Fälle nicht überdeckt wird, ist sehr schmal, und die in der Lösung vorkommenden Parameter hängen explizit nicht vom Radius der Kugel oder von deren Geschwindigkeit ab. Weiterhin wird gezeigt, daß (im Bereich der bis jetzt vorliegenden Experimente) die Zeitabhängigkeit der Vorgänge in Dampf und Kühlmittel vernachlässigbar ist und deshalb eine quasi-stationäre Approximation für dieses Gebiet zulässig ist. Daher kann die aus der stationären Betrachtung erhaltene Theorie zur Behandlung des instationären Wärmeübergangs an der Kugel verwendet werden. Die Lösung dieses Problems wird angegeben.

УСТОЙЧИВОЕ И ПЕРЕХОДНОЕ ПЛЕНОЧНОЕ КИПЕНИЕ НА СФЕРЕ ПРИ НАЛИЧИИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация — Приводится теоретическое исследование пленочного кипения на сфере при наличии вынужденной конвекций. Прежде всего анализируется стационарный процесс, т. е. делается допущение о постоянстве температуры сферы. В модель включен также недогрев жидкости. Получены простые аналитические решения для толщины пленки пара и локального числа Нуссельта для двух предельных случаев, грубо соответствующих наличию большого и малого недогрева. Полученные результаты представляют особую ценность благодаря следующим двум факторам: область, не охваченная этими предельными случаями, довольно мала и входящие в решения параметры явно не зависят от радиуса сферы или ее скорости. Кроме того показано, что для параметров, соответствующих проведенным экспериментам, нестационарные явления в паре и жидкости пренебрежимо малы и поэтому для этих областей справедливо квазистационарное приближение. Таким образом, стационарная теория может быть использована для формулировки задачи нестационарного теплообмена сферы. Приведено решение данной задачи.